For the rest of geometry, the treatment is not so detailed. The general topics of the circle, similar figures, methods of attack for theorems and problems, short cuts for the teacher, are given. There is a chapter on limits which gives some very good suggestions in regard to teaching that subject.

For solid geometry, the treatment is confined to a discussion of the aim of the subject, the use of models, and methods of drawing figures. A chapter on applied problems, in which the author takes conservative ground, completes the treatment of geometry.

There are two points of criticism in regard to the discussion of geometry. Little emphasis has been put on the use of algebra in geometry. In view of the fact that algebra is taught before geometry, a more extended use of algebraic methods would seem wise. Again, the geometry course is necessarily limited in time and if more originals are taught, the text must give way. New teachers hardly know what to omit, and an outline of important propositions, if only for the first book, would be valuable, such, for instance, as is proposed by the National Committee of Fifteen on Geometry Syllabus.

The treatment of algebra is much more limited. After discussing what topics should be taught, the author takes up the methods of presenting the difficult ones, such as negative and irrational numbers. There is also a treatment of the use of graphs in the solution of problems, and a discussion of the use of logarithms. A chapter on the use of trigonometry by high school students completes the book.

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Higher Algebra for Colleges and Secondary Schools. By CHARLES DAVISON, Sc.D., Mathematical Master at King Edward's School, Birmingham. Cambridge: at the University Press, 1912. vi+320 pages. $2.00.

The introduction to this book is in part a review of some of the ground covered in the author's *Algebra for Secondary Schools.* We find here four chapters, treating: the binomial theorem for positive integral index, product and polynomial theorems, partial fractions, and complex quantity.

The first chapter contains chiefly examples and exercises which bring out important properties of the binomial coefficients, such as the following: (1) Show that $(n+1)_nC_0 - n_nC_1 + (n-1)_nC_2 - \cdots (-1)^n{}_nC_n = 0$; (2) Show that the integral part of $(a + \sqrt{a^2-1})^n$ is odd if n be a whole number. Here evidently a also should be a whole number.

The second and third chapters treat briefly in twelve pages the other subjects mentioned above, and include four lists of exercises. The latter are of a distinctly higher type than those found in American texts on College Algebra, yet are within reach of students above the freshman year. The same is true generally of the large number of interesting and instructive lists of exercises scattered through the text.

The treatment of complex quantity in Chapter IV is geometric, the vector OP being represented by one of the symbols (a, α) or (ρ, θ). Addition is based on the parallelogram, which includes the construction for the sum of real numbers.

"To multiply one complex number (a, α) by another (b, β), the number (a, α) must be treated in the same way that unity is treated to obtain (b, β)." Following this out geometrically gives $(a, \alpha) \times (b, \beta) = (ab, \alpha + \beta)$. Then i turns out to be the unit vector at right angles to the initial line, and this leads to $(\rho, \theta) = x+yi$. The chapter closes with a discussion of the commutative and associative laws, and of the nth roots of unity, followed by a list of exercises.

Part II contains six chapters dealing with convergency and divergency of series, binomial theorem, any index, exponential series, logarithmic series, summation of series, recurring series.

While the selection of the material here is excellent and well adapted to interest the student, the treatment is rather loose in places. On page 38 the sum of the geometric series is given as the sum of its first n terms. Several errors are due to statements about series in general which are true only for series of positive terms. Thus on page 42: "The ratio $u_1 + u_2 + u_3 + \cdots : v_1 + v_2 + v_3 + \cdots$ lies between the least and greatest of the ratios $u_1 : v_1$, $u_2 : v_2$, $u_3 : v_3$, $\cdots$." Nothing is said about the signs of the u's and v's. The same statement occurs on page 121.

The treatment of the binomial theorem, any index, in Chapter VI extends only to rational indices. This chapter also contains Vandermonde's theorem, two theorems on homogeneous products, and a considerable number of exercises.

In Chapter VII on exponential series, e is defined by means of the usual series, and is then computed to four places by using the first ten terms. It is not shown that the omitted terms could not affect the fourth place. Then $f(m)$ is defined as the exponential series in m, and it is shown that $f(m) \cdot f(n) = f(m + n)$, and from this that $f(x) = e^x$ for rational values of x. Other values of x are not considered.

The logarithmic series in Chapter VIII is derived by aid of the binomial theorem and hence is obtained only for rational values of x. In computing log 2 to four places, it is again not shown that the remainder of the series could not affect the fourth place. A number of interesting exercises on logarithms are given; the last one calls for evaluating the limit of the expression which leads to Euler's constant.

Under summation of series in Chapter IX we find the sums of some special series, definitions of the polygonal and figurate numbers, and the method of summation by differences, including numerous worked examples and exercises. Recurring series are briefly treated in Chapter X.

Part III contains four chapters, treating of inequalities and maxima and minima, approximations, limits and differential coefficients, convergency and divergency of series (second treatment).

In the first of these chapters occur a number of "important inequalities," several of which require restrictions that are not stated. Thus we find: "If $a > b$ then $a^m > b^m$ when m is a positive integer." Again: "If $a > x$, $b > y$, $c > z$, then $abc > xyz$." In the work on inequalities which follows, more care is taken to make the statements accurate. We have here comparisons of the arithmetic and geometric means of n quantities, and of the mean of the mth

powers with the mth power of the mean of n quantities. Under maxima and minima we find for example: "The maximum of $x^a y^b z^c$, when $x + y + z = C$."

The chapter on approximations consists chiefly of many instructive examples and exercises. In the next chapter we find the usual theorems on limits, with the usual intuitive proofs. This is followed by a treatment of some indeterminate forms. Then the derivative is defined and polynomials and products of linear factors are differentiated. The last exercise of the chapter rather strains the use of the equality sign.

Chapter XIV concludes the treatment of convergency and divergency of series. Additional tests are given, a direct comparison test, Raabe's test, and a logarithmic test. In the proofs of all of these, the necessary restrictions on the signs of the terms are not stated.

Part IV contains eight chapters treating the relations between coefficients and roots, transformation of equations, properties of equations, equal and commensurable roots, cubic and biquadratic equations, location of roots (Sturm's theorem), Horner's method, and determinants. The presentation of this material is clear and direct, and is accompanied by numerous well-chosen examples and exercises.

One or two slips should be noted. The theorem on the number of roots, page 152, states more than is proved, since the existence of a root is not established. On page 177, under Rolle's Theorem, is the rather remarkable statement: "It follows that every root of the equation $f'(x) = 0$ lies between a pair of successive roots of the equation $f(x) = 0$." Similarly on page 185 under equal roots we find: "If the roots of $f(x) = 0$ be all different, all the roots of the equation $f'(x) = 0$ lie between successive pairs of roots of the given equation."

Chapter XXII develops the principal properties of determinants, the elements of a determinant being obtained by permutation of the subscripts in the principal term. Application is made to solution of equations and to elimination, the latter including Bezout's method.

Part V contains five chapters relating to continued fractions, recurring continued fractions, continued fractions and series, theory of numbers and indeterminate equations. In each case a few elementary theorems are demonstrated, and these are followed by numerous worked examples and lists of exercises.

This concludes the body of the text, 279 pages. The next thirty pages contain twenty topics for short essays and fifteen "problem papers." The book closes with a list of answers.

Aside from such defects as those noted, the presentation of the material in the book is excellent. Perhaps two thirds of its pages are devoted to worked examples and exercises, and these are of such variety that they would make an interesting collection for any teacher of college mathematics.

The book seems to be quite free from typographical errors. Only three were noticed: on page viii, line 3, the radical sign should be deleted; on page 140, line 7, a transposition is needed; on page 179, line 7, a fraction line is missing. To the list of symbols should be added the following: Σ, $\sim$, $\not<$, and $\not>$.

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